

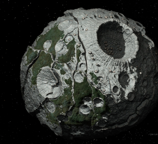
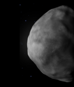
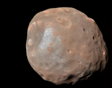

Using effective density spectrum to constrain crustal density profile of Vesta and Ceres

Anton I. Ermakov[1], Ryan S. Park[1] , Carol A. Raymond[1] , Julie C. Castillo-Rogez[1] , Christopher T. Russell[2]

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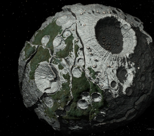
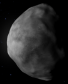
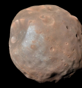
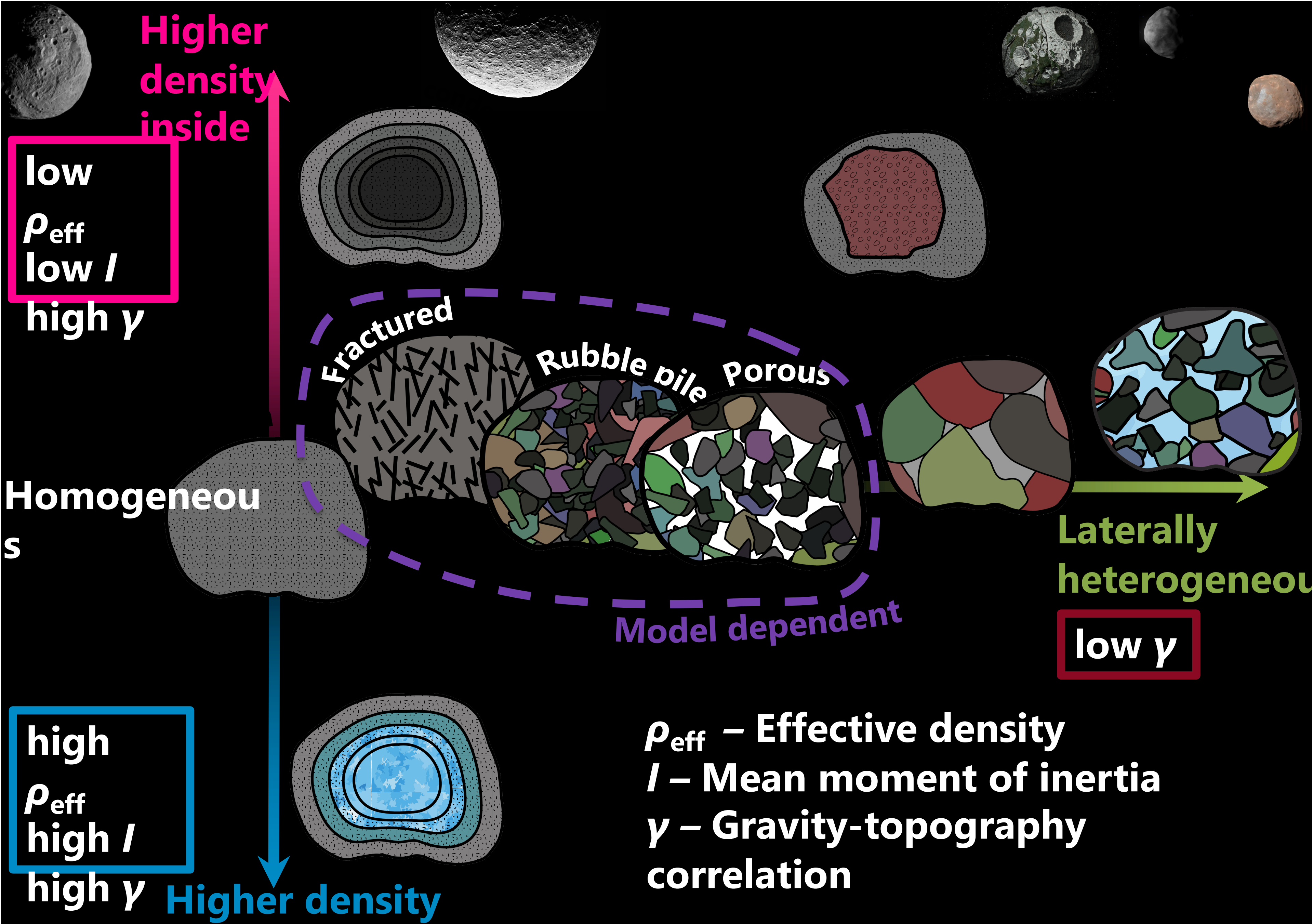
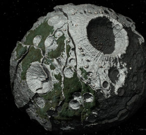
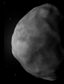
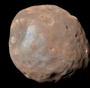




Outline

- Gravity and topography in spherical harmonics
- Derivation of effective density spectrum
- Effective spectrum of Vesta
- Why I thought it would work for Ceres
- and why it does not.


Higher density inside
low ρ_{eff}
low I
high γ
Fractured
Rubble pile
Porous
Homogeneous
Model dependent
Laterally heterogeneous
low γ
high ρ_{eff}
high I
high γ
Higher density outside
ρ_{eff} – Effective density
I – Mean moment of inertia
γ – Gravity-topography correlation



Spherical Harmonics

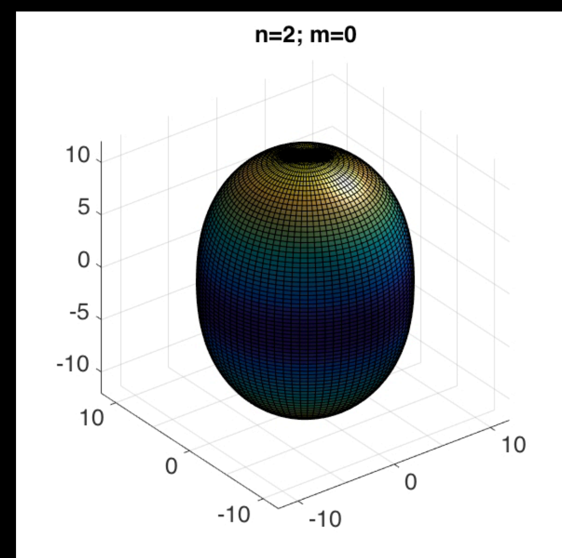
- **Shape**

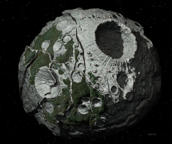
$$r(\phi,\lambda) = R_0 \sum_{n=0}^{\infty} \sum_{m=-n}^{n} A_{nm} Y_{nm}(\phi,\lambda)$$

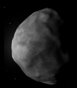
- **Gravitational potential**

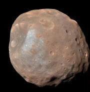
$$U(r,\phi,\lambda) = \frac{GM}{r} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left(\frac{R_0}{r}\right)^n C_{nm} Y_{nm}(\phi,\lambda)$$

U – gravitational potential
ϕ – latitude
λ – longitude
r – radial distance
n – degree
m – order









Spherical Harmonics

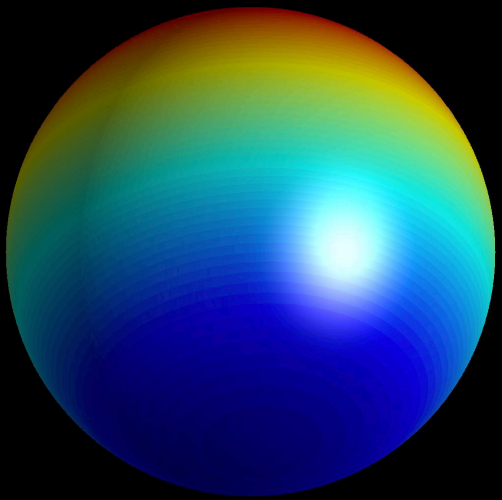
$$r(\phi,\lambda) = R_0 \sum_{n=0}^{\infty} \sum_{m=-n}^{n} A_{nm} Y_{nm}(\phi,\lambda)$$

$$U(r,\phi,\lambda) = \frac{GM}{r} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left(\frac{R_0}{r}\right)^n C_{nm} Y_{nm}(\phi,\lambda)$$

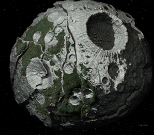
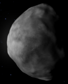
- **RMS spectrum**

$$M_n^{gg} = \sqrt{\frac{\sum_{m=-n}^{n} C_{nm}^2}{2n+1}}$$

$$M_n^{tt} = \sqrt{\frac{\sum_{m=-n}^{n} A_{nm}^2}{2n+1}}$$





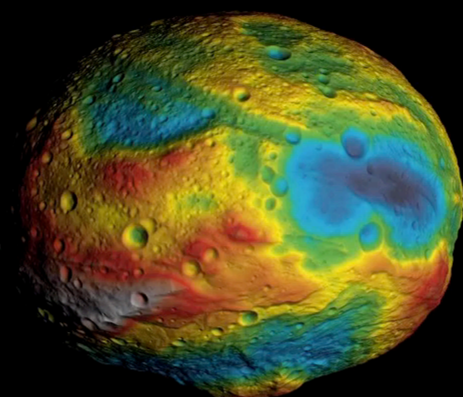
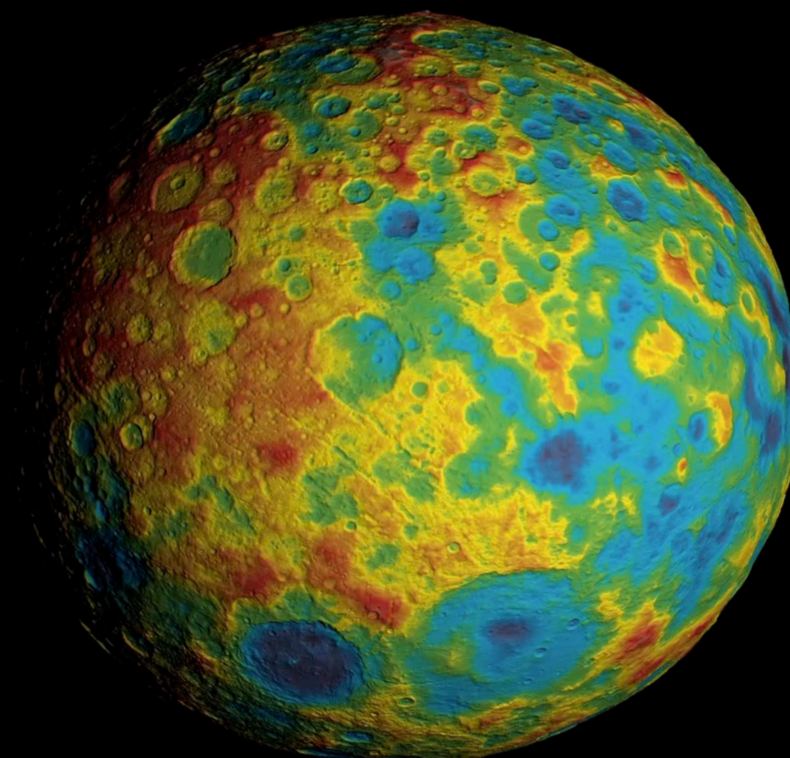

Dawn *geophysical* data

- Shape model
  - Stereophotogrammetry (SPG) from DLR
  - Stereophotoclinometry (SPC) from JPL
  - Mutually consistent with the accuracy much better than the spatial resolution of gravity field

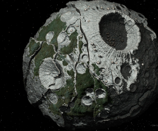
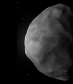
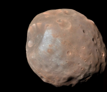
- Gravity field
  - Accurate up to n = 18 (λ=93 km) for Vesta (Konopliv et al., 2014)
  - Globally accurate up to n = 18 (λ=174 km) for Ceres (Konopliv et al., 2017)
  - Locally accurate up to n = 59 (λ=50 km) (Park et al., in preparation)



Vesta and Ceres

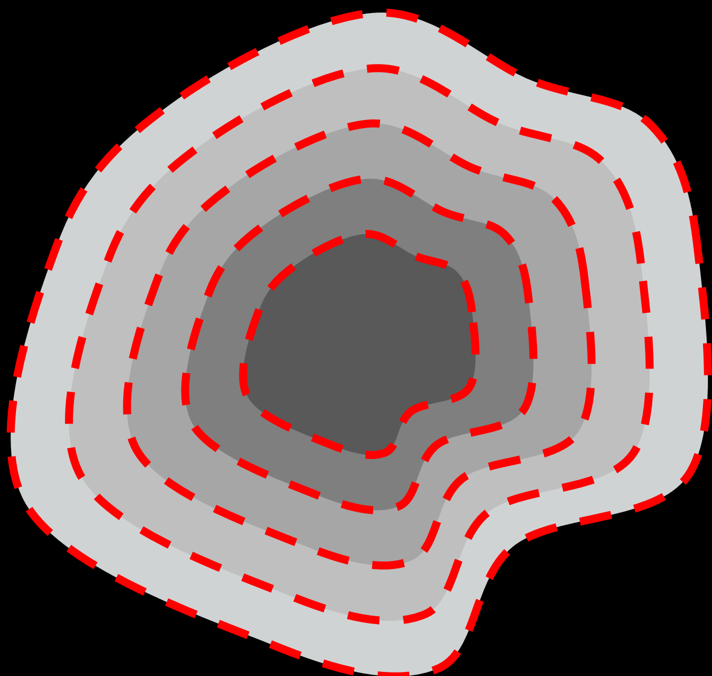


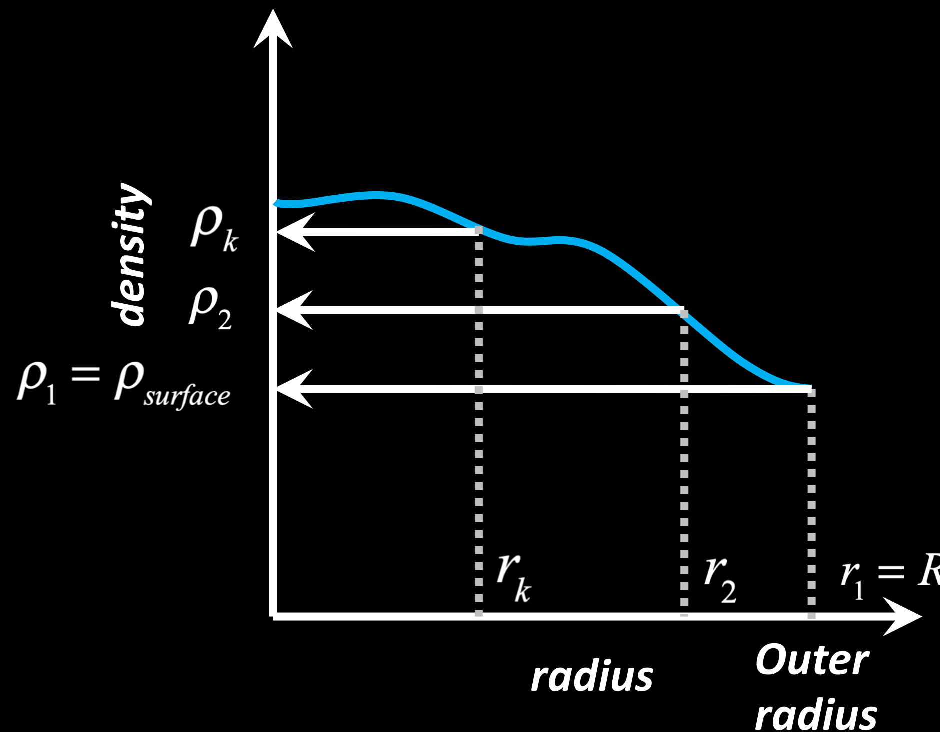




Effective density

$$\sigma_{nm}^{const} = \{\bar{C}_{nm}^{const}, \bar{S}_{nm}^{const}\}$$

Gravity spherical harmonic coefficients referenced to the volumetric radius of a layer





Contribution of *k*-th layer to total gravity:

$$\sigma_{nm}^{const} \cdot \underbrace{\frac{4}{3}\pi r_k^3(\rho_k - \rho_{k-1}) \cdot \frac{1}{M}}_{\text{Fractional mass of a layer}} \cdot \underbrace{\left(\frac{r_k}{R}\right)^n}_{\text{Upward continuation factor}}$$



Effective density

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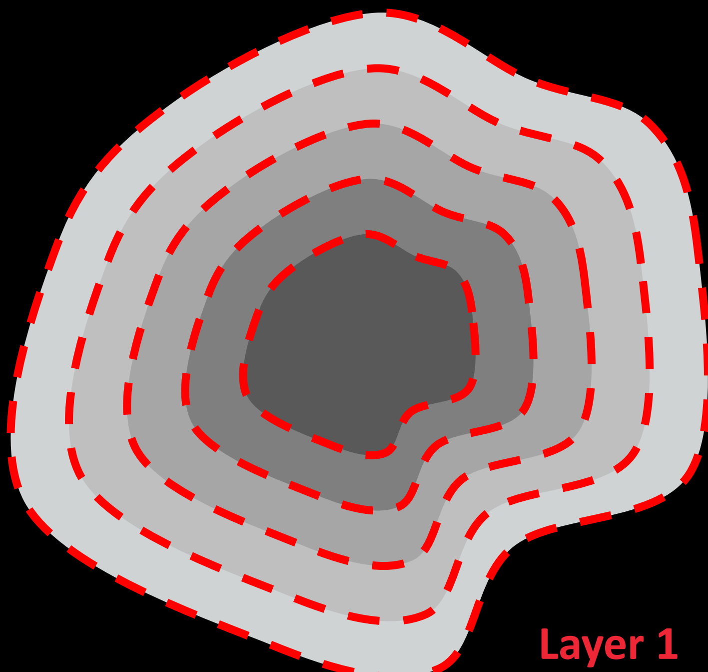
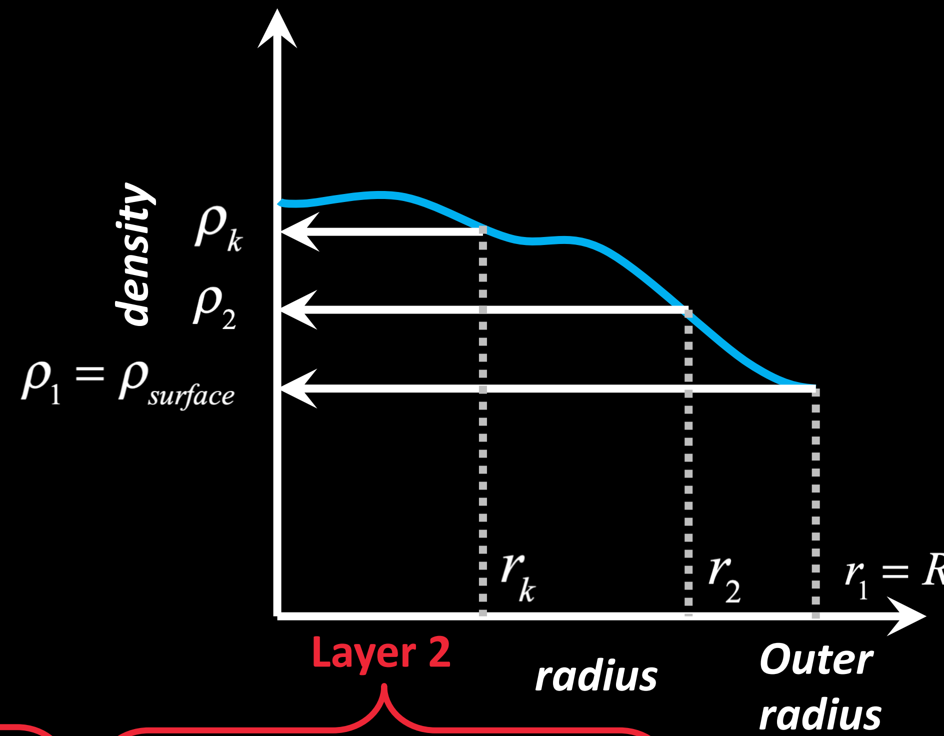




$$\sigma_{nm} = \underbrace{\frac{1}{M}\sigma_{nm}^{const}\frac{4}{3}\pi r_1^3\rho_1\left(\frac{r_1}{R}\right)^n}_{\text{Layer 1}} + \underbrace{\frac{1}{M}\sigma_{nm}^{const}\frac{4}{3}\pi r_2^3(\rho_2-\rho_1)\left(\frac{r_2}{R}\right)^n}_{\text{Layer 2}} + \dots$$

$$\underbrace{\dots + \frac{1}{M}\sigma_{nm}^{const}\frac{4}{3}\pi r_k^3(\rho_k-\rho_{k-1})\left(\frac{r_k}{R}\right)^n}_{\text{Layer k}}$$

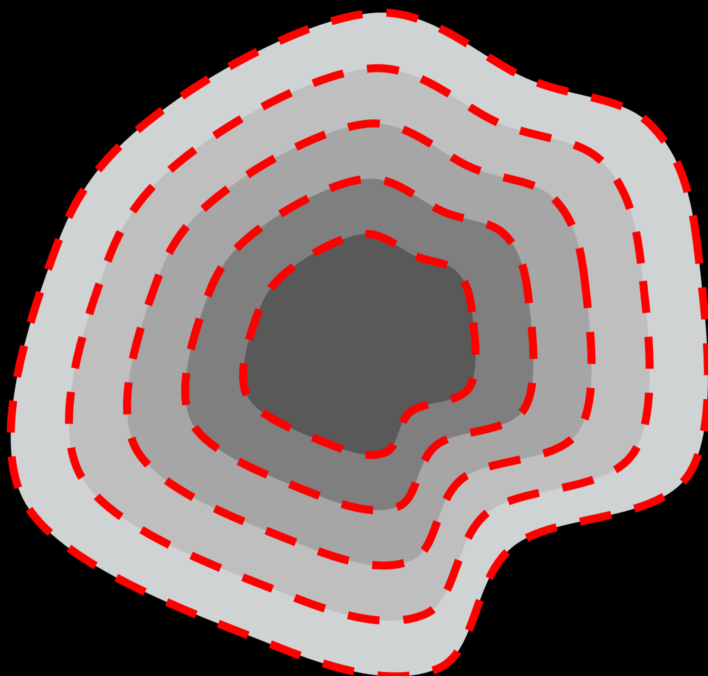
$$\Rightarrow \sigma_{nm} = \frac{\sigma_{nm}^{const}}{\bar{\rho}R^{n+3}}\int_{R^+}^{0} r^{n+3}\frac{d\rho(r)}{dr}dr$$

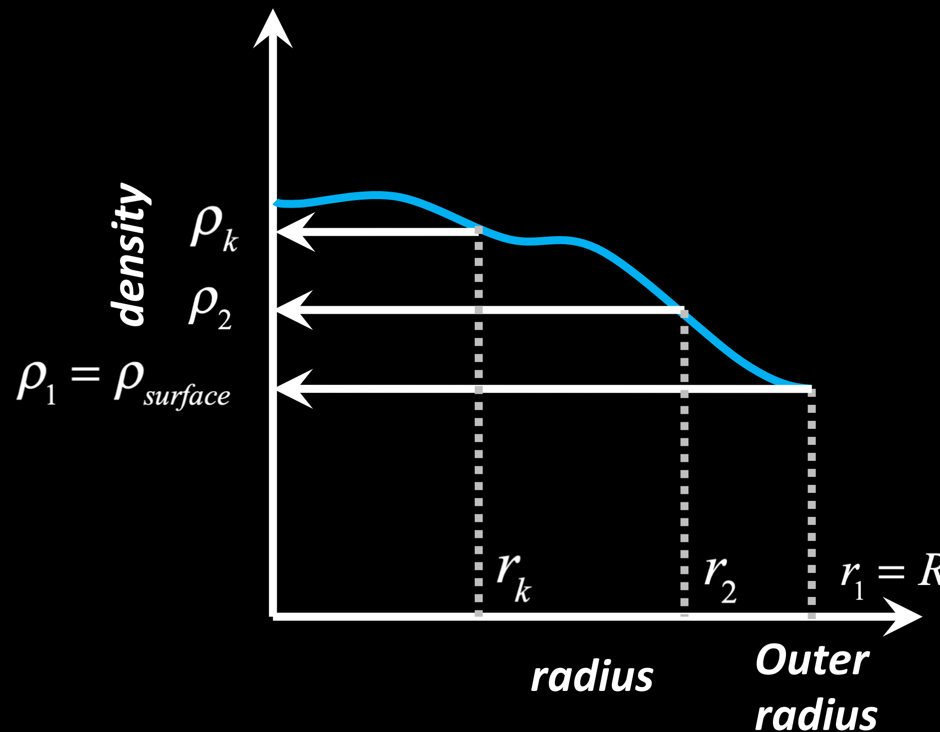


Effective density

$$\sigma_{nm}^{const} = \{\bar{C}_{nm}^{const}, \bar{S}_{nm}^{const}\}$$

Gravity spherical harmonic coefficients referenced to the volumetric radius of a layer

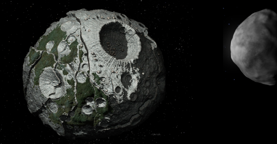


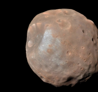


***Def*: effective density**

$$\tilde{\rho}_n = \frac{\sigma_{nm}}{\sigma_{nm}^{const}} \bar{\rho}$$

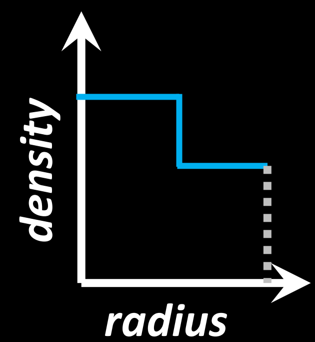
$$\boxed{\tilde{\rho}_n = \rho_{surface} + \frac{1}{\bar{\rho} R^{n+3}} \int_{R^-}^{0} r^{n+3} \frac{d\rho(r)}{dr} dr}$$



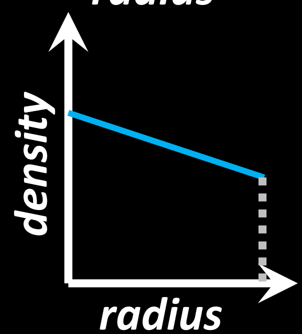


Simple density profiles

$$\boxed{\tilde{\rho}_n = \rho_{surface} + \frac{1}{R^{n+3}} \int_{R^-}^{0} r^{n+3} \frac{d\rho(r)}{dr} dr}$$

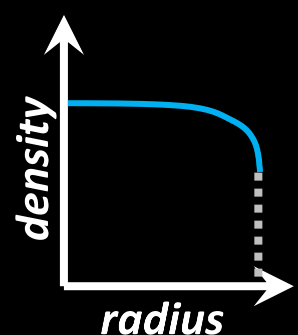


$$\tilde{\rho}_n = \rho_{upper} + \left(\frac{r_{lower}}{r_{upper}}\right)^{n+3} \left(\rho_{upper} - \rho_{lower}\right)$$



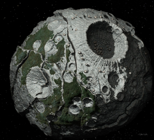
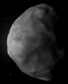
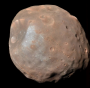
$$\tilde{\rho}_n = \frac{n\rho_{surface} + 4\bar{\rho}}{n+4}$$

$$\tilde{\rho}_n = \frac{4n\bar{\rho} + 12\bar{\rho} - n\rho_{center}}{3(n+4)}$$



$$\tilde{\rho}_n = \rho_{surface} + (-1)^n \left(\frac{r}{R}\right)^{n+3} \Delta\rho e^{-R/d} \left[\Gamma(n+4) - \Gamma(n+4, -R/d)\right]$$



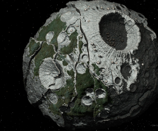
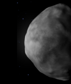
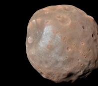
Curious properties of effective density

$$\boxed{\tilde{\rho}_n = \rho_{surface} + \frac{1}{R^{n+3}} \int_{R^-}^{0} r^{n+3} \frac{d\rho(r)}{dr} dr}$$

$$\tilde{\rho}_0 = \bar{\rho}$$

$$\tilde{\rho}_\infty = \rho_{surface}$$



Spatio-spectral concentration

$$\lambda = \frac{\|g\|_R^2}{\|g\|_\Omega^2} = \frac{\int_R g^2 \, d\Omega}{\int_\Omega g^2 \, d\Omega} = \max$$

Spatial localization

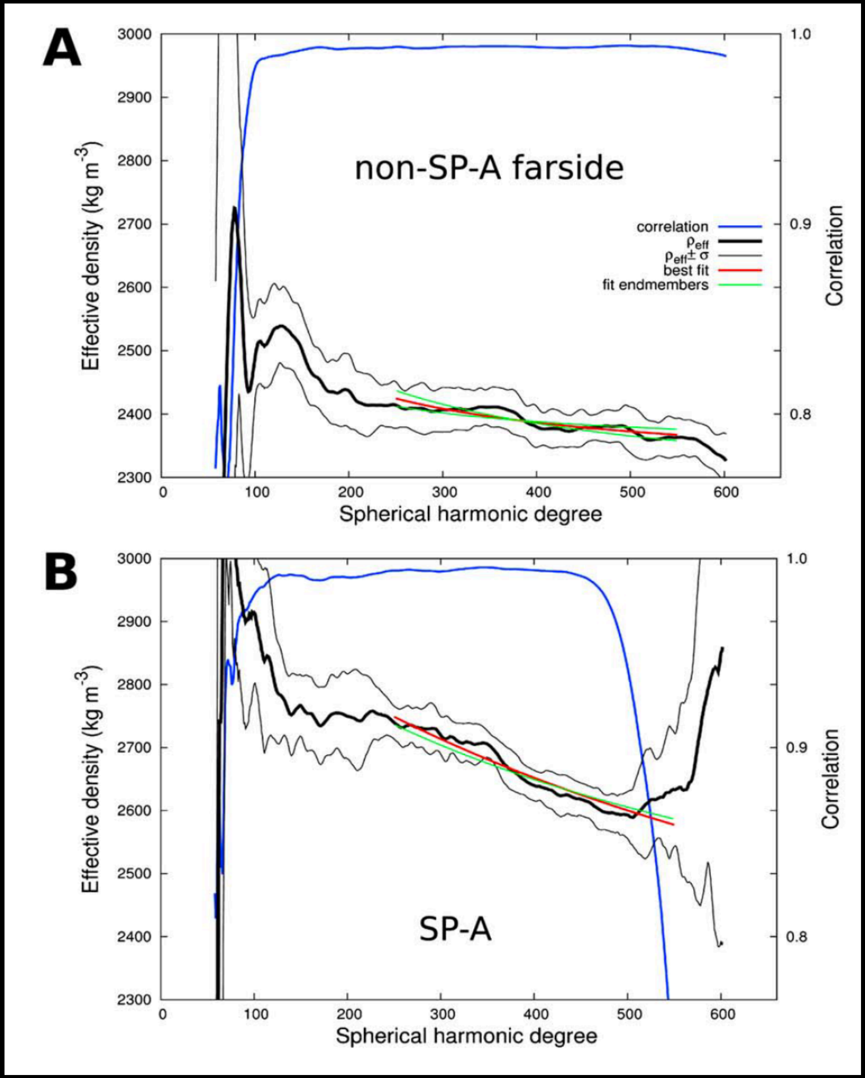
$$\lambda = \frac{\|g\|_L^2}{\|g\|_\infty^2} = \frac{\sum_{l=0}^{L} \sum_{m=-l}^{l} h_{lm}^2}{\sum_{l=0}^{\infty} \sum_{m=-l}^{l} h_{lm}^2} = \max$$

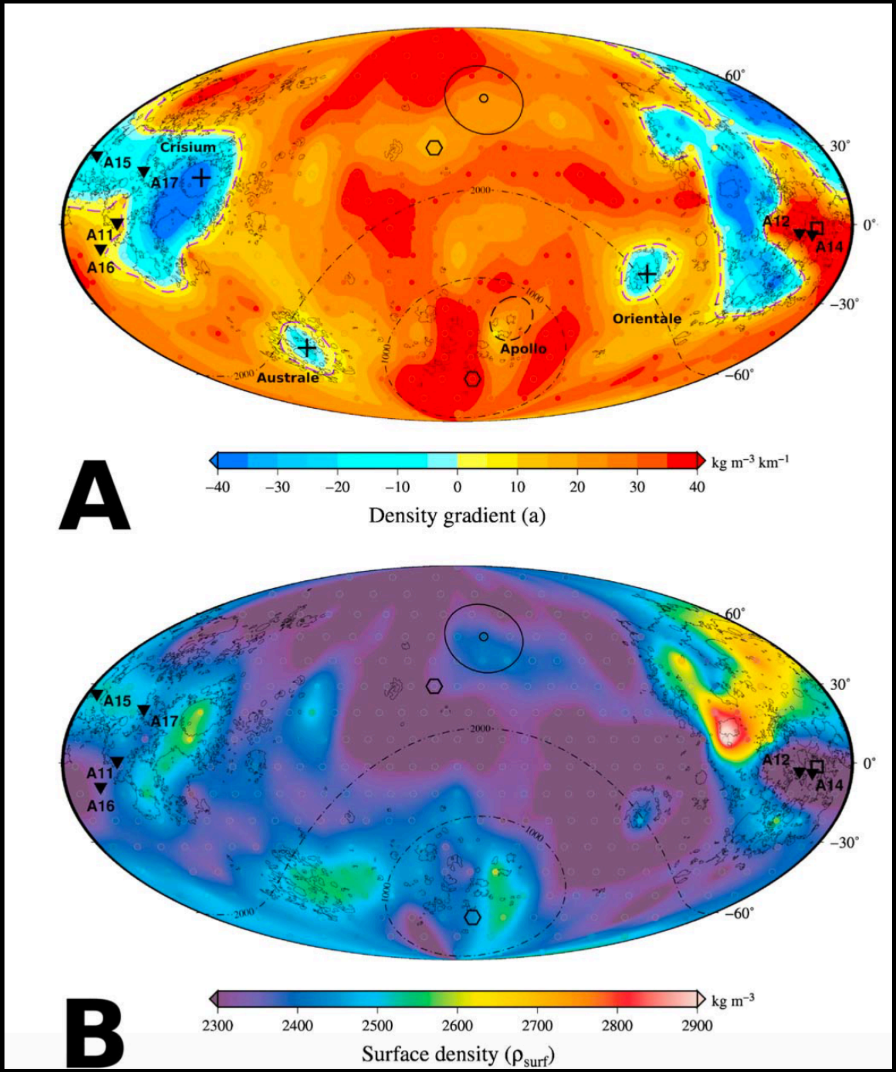
Spectral localization

R **– region of localization**
λ **– eigenvalue = concentration factor**

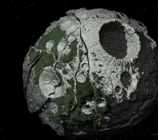
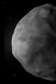
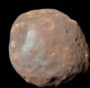
- **Simons et al., 2006 presents a way to find the eigenfunctions and shows that they are identical**
- **Allows localized estimates of crustal density**

Effective density with GRAIL

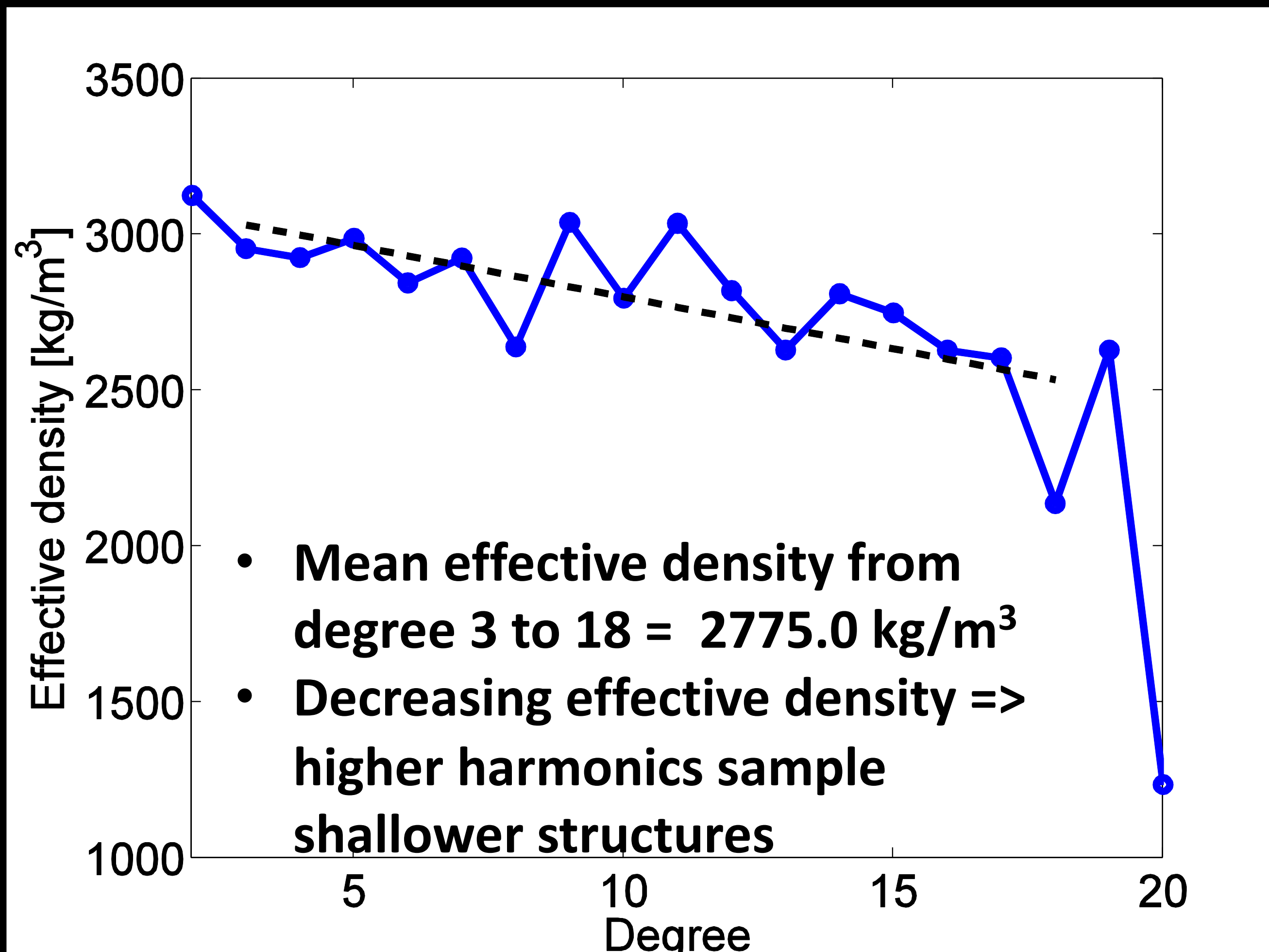






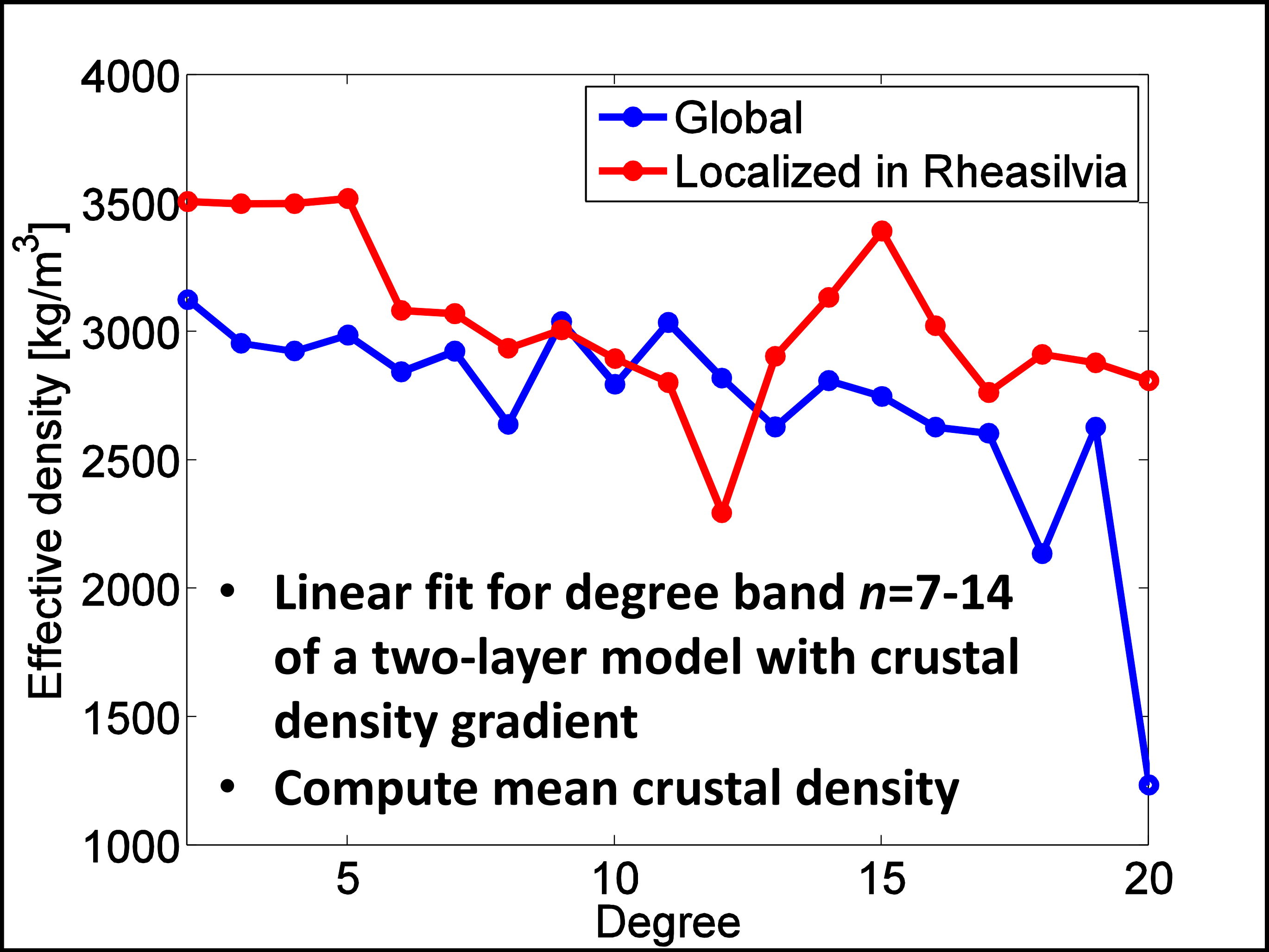




Vesta global effective density



- Mean effective density from degree 3 to 18 = 2775.0 kg/m^3
- Decreasing effective density => higher harmonics sample shallower structures

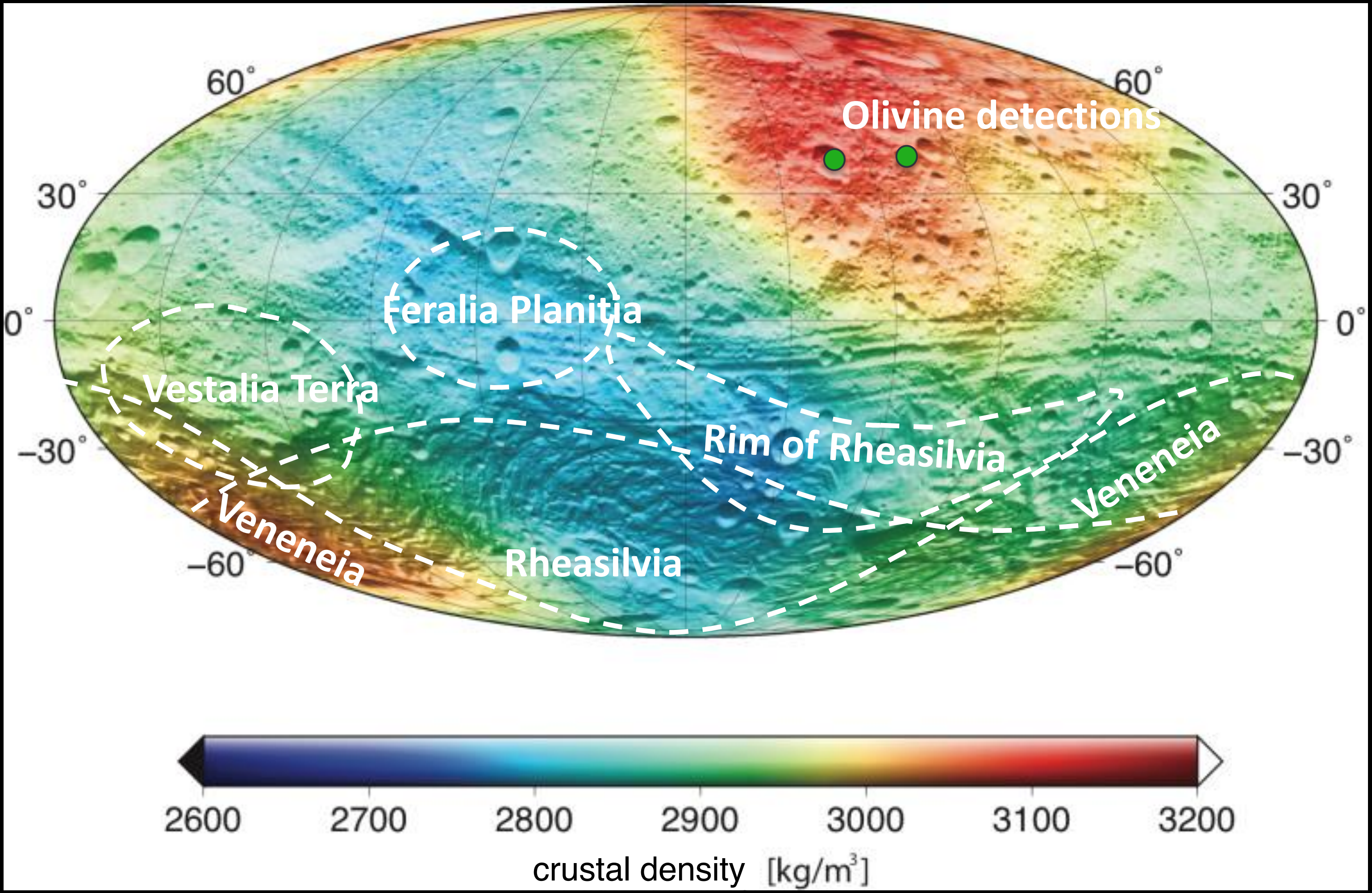
Effective density linear fit

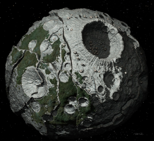
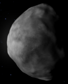
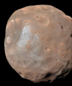


- Linear fit for degree band *n*=7-14 of a two-layer model with crustal density gradient
- Compute mean crustal density

Interpolated crustal density

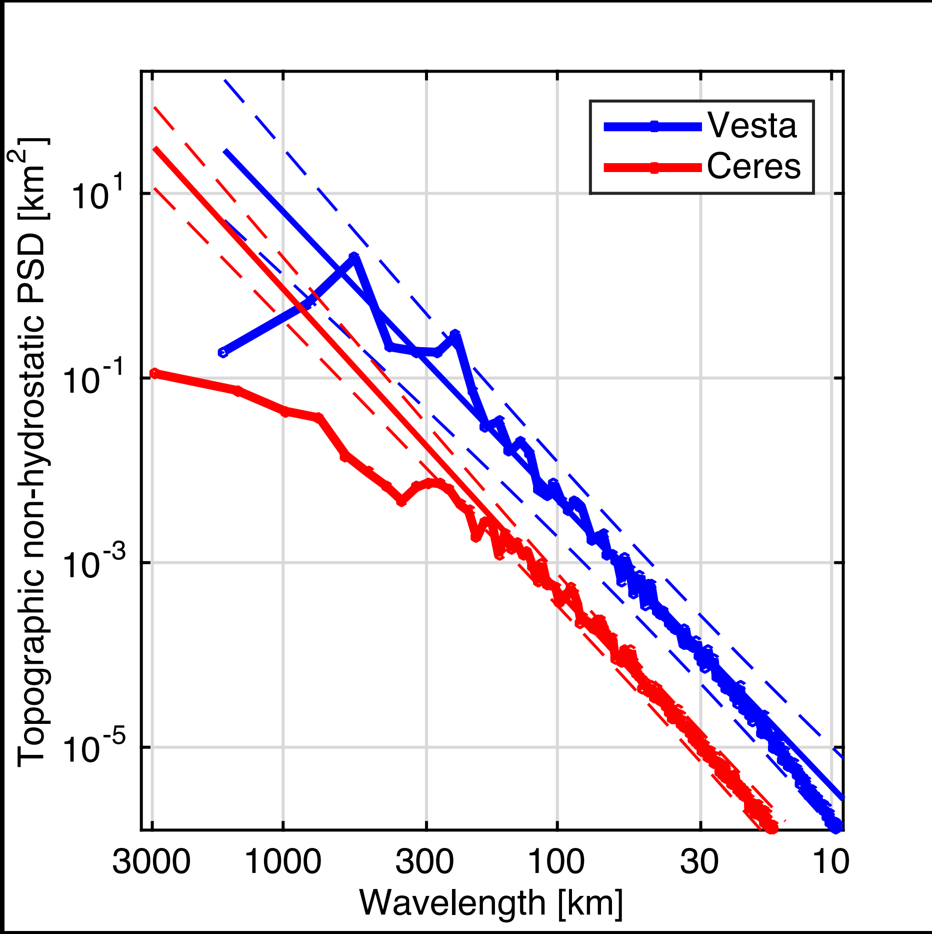
Bandwidth *L* = 5, window = 50° spherical cap



Evidence for viscous relaxation

- **More general approach: <u>study topography power spectrum</u>**
- **Power spectra for Vesta closely fits with the power law to the lowest degrees (λ < 750 km)**
- **Ceres power spectrum deviates from the power law at λ > 270 km**



Ermakov et al., 2017a

Evidence for viscous relaxation

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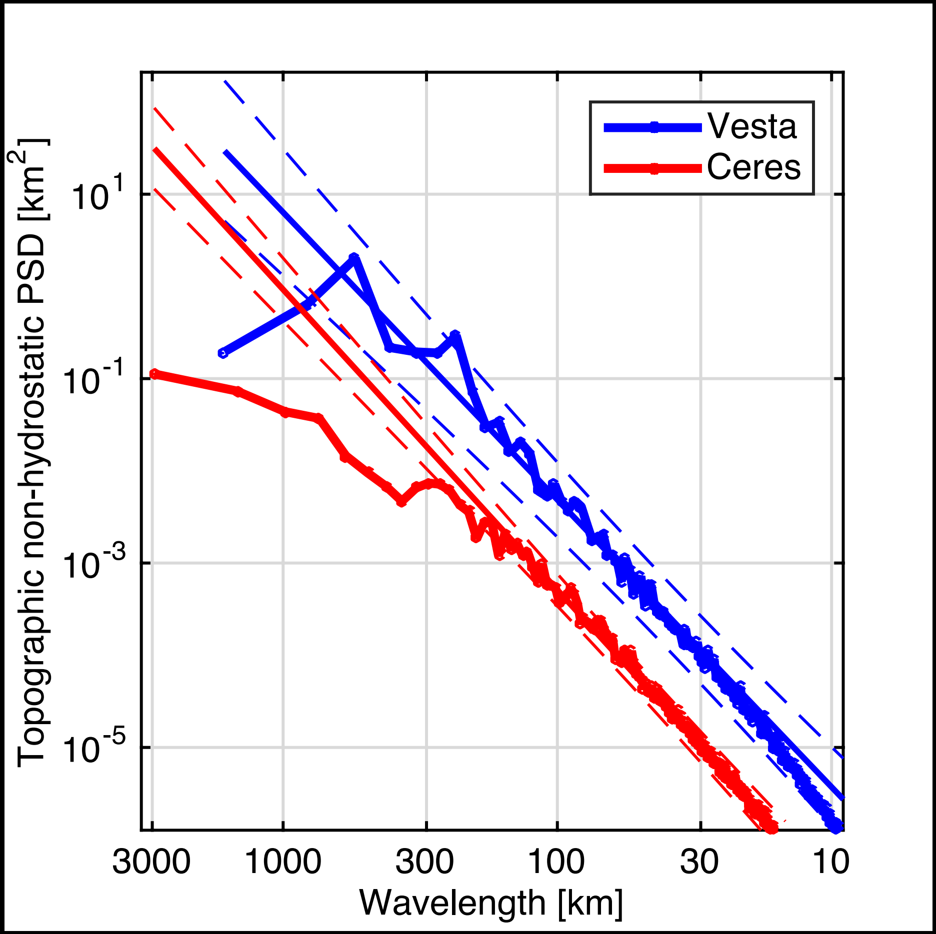

Ermakov et al., 2017a

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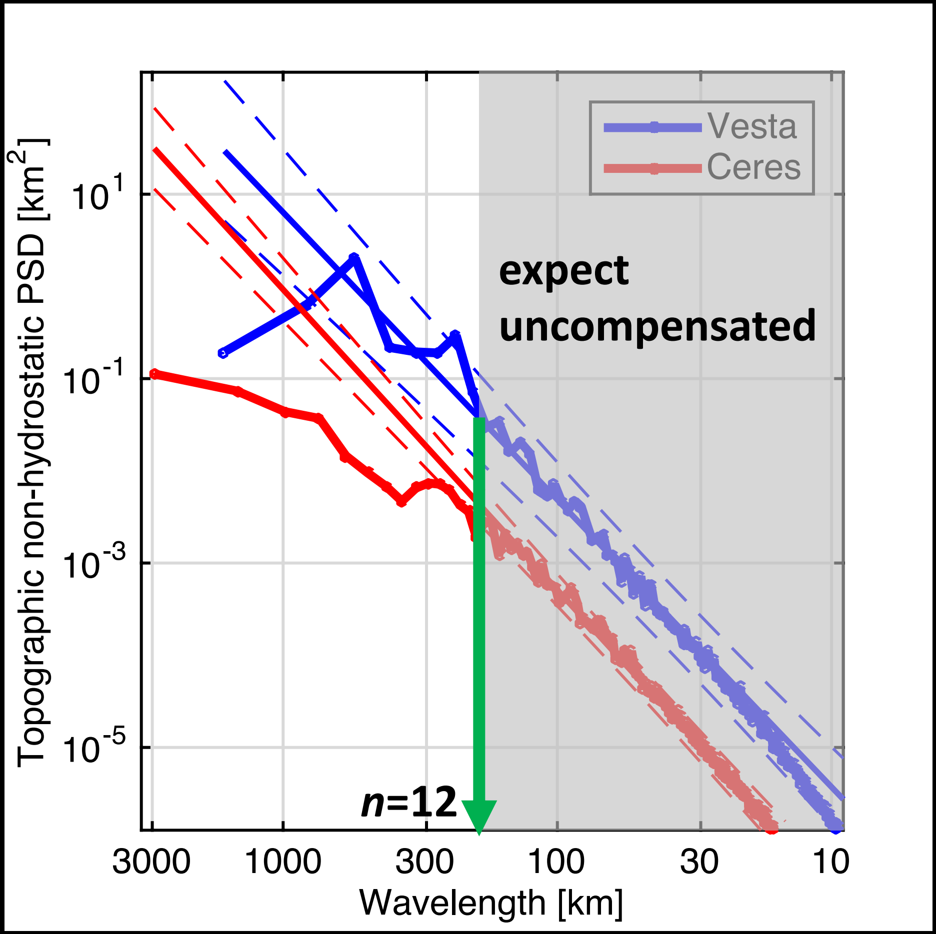

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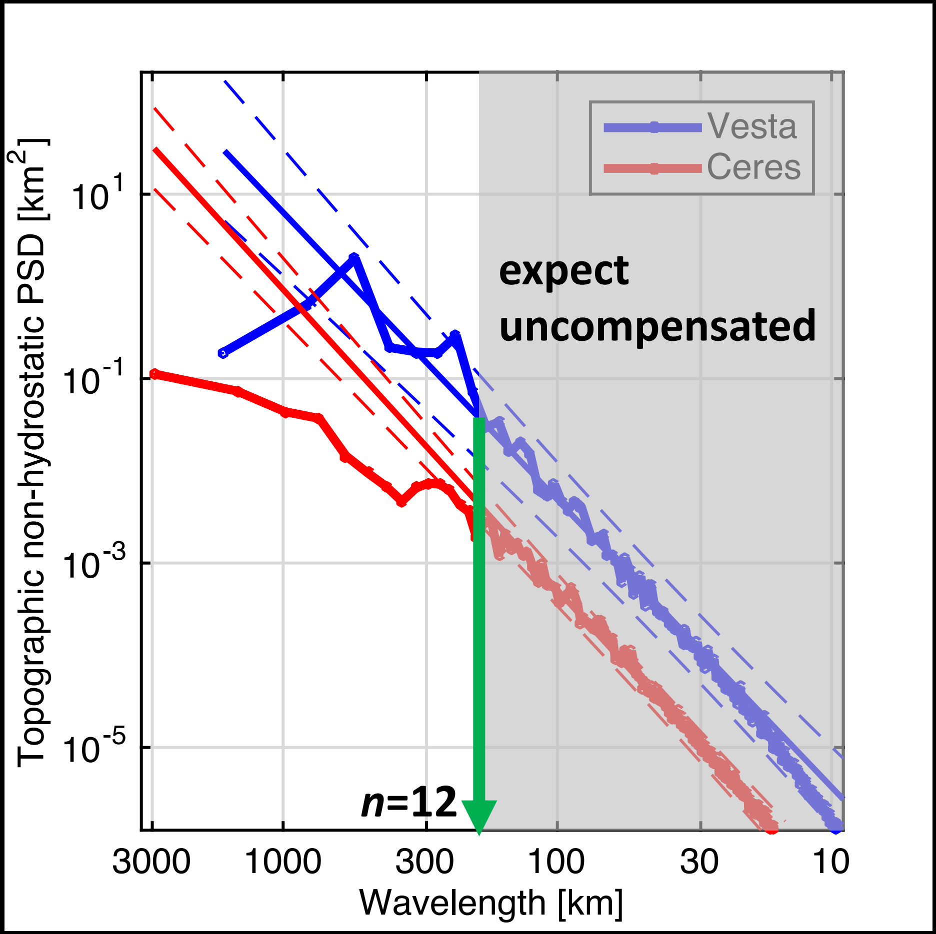
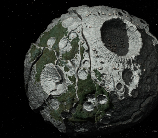
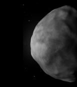
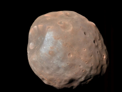


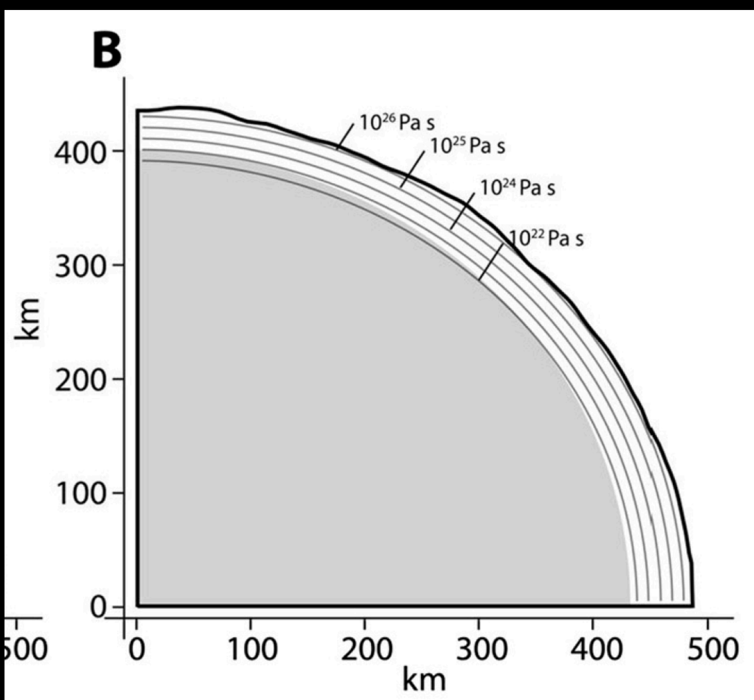
Ermakov et al., 2017a

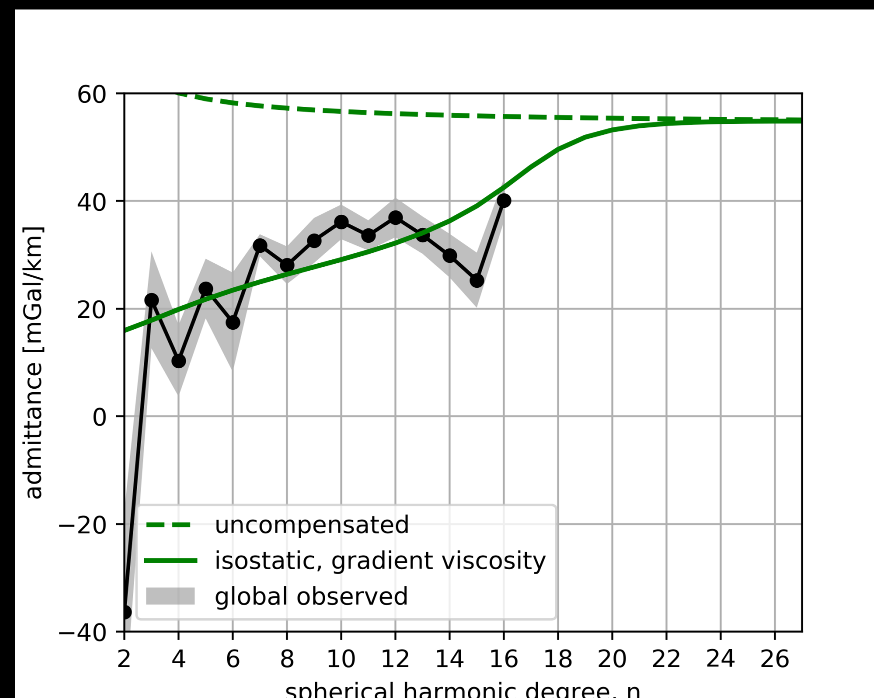


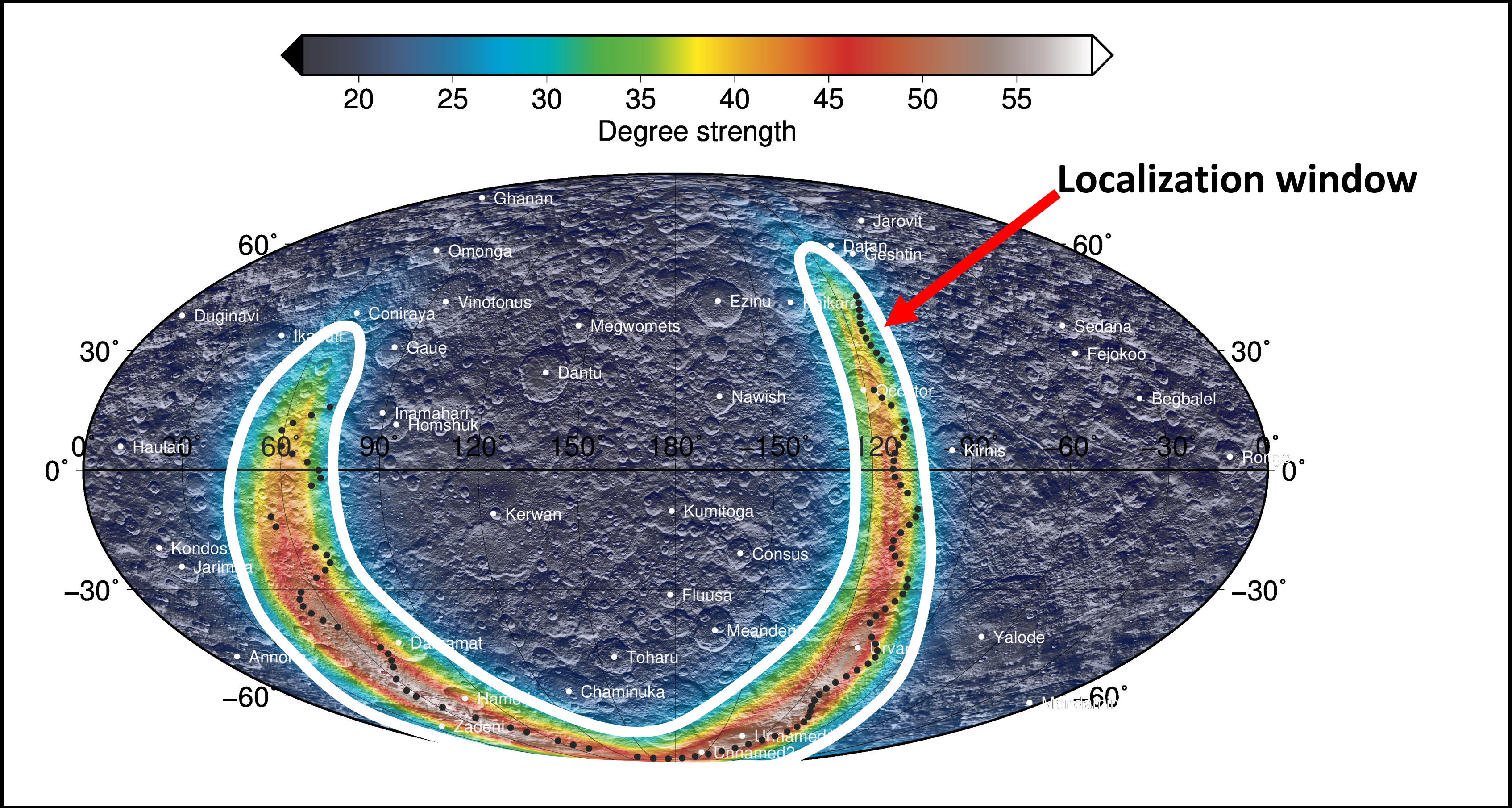
Ceres effective density why I expected it would work

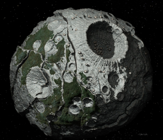
Viscosity profile inferred by Fu et al., 2017 from topography relaxation

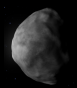


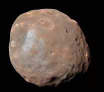


Degree strength from Dawn's second extended mission









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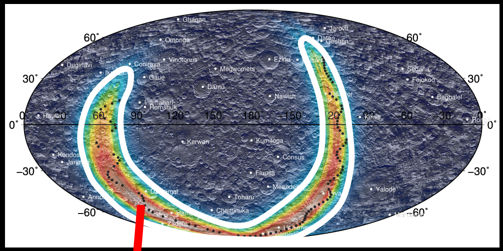
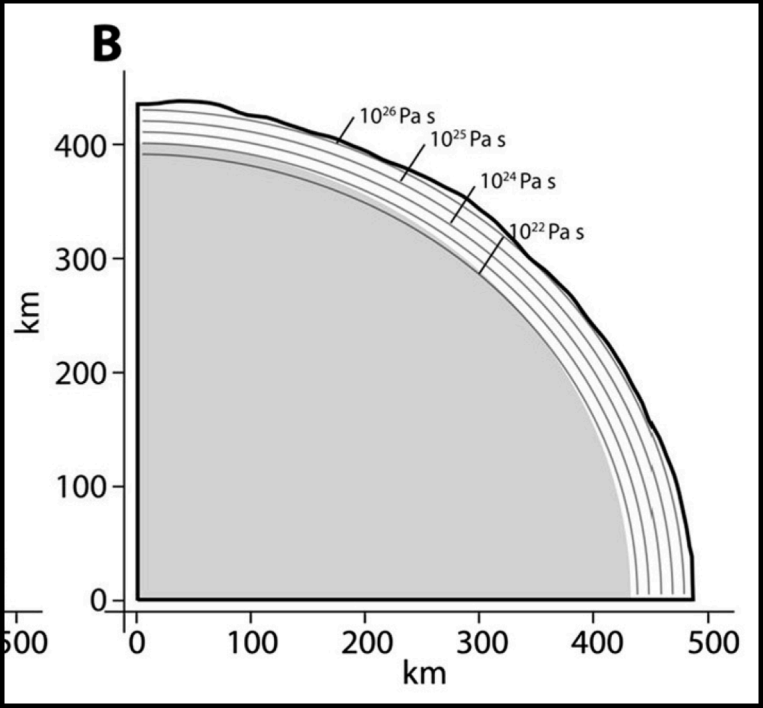
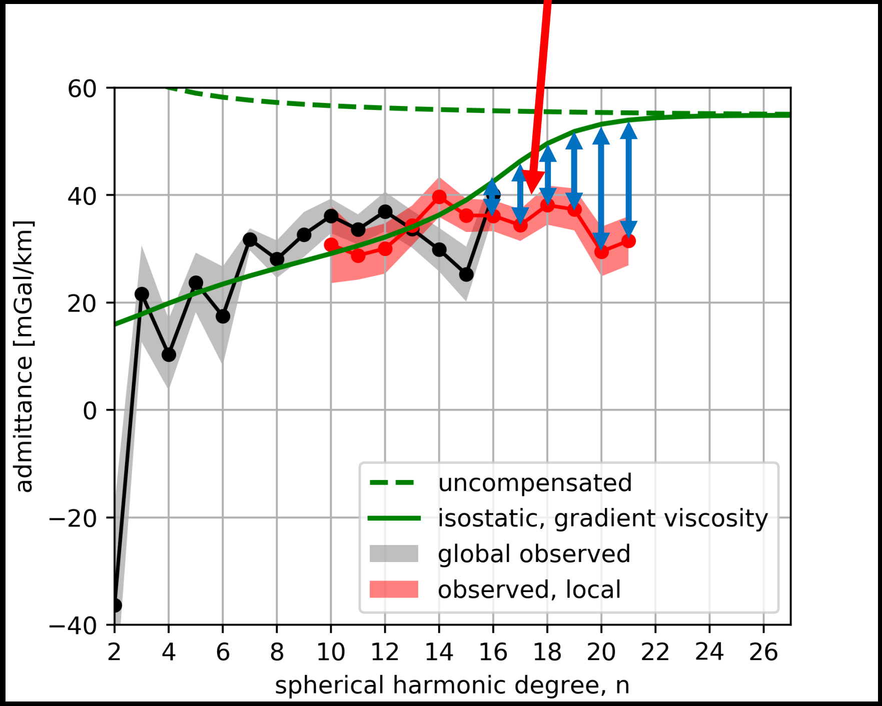
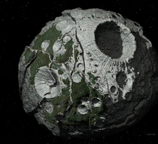
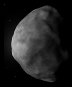
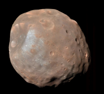


Viscosity profile inferred by Fu et al., 2017 from topography relaxation







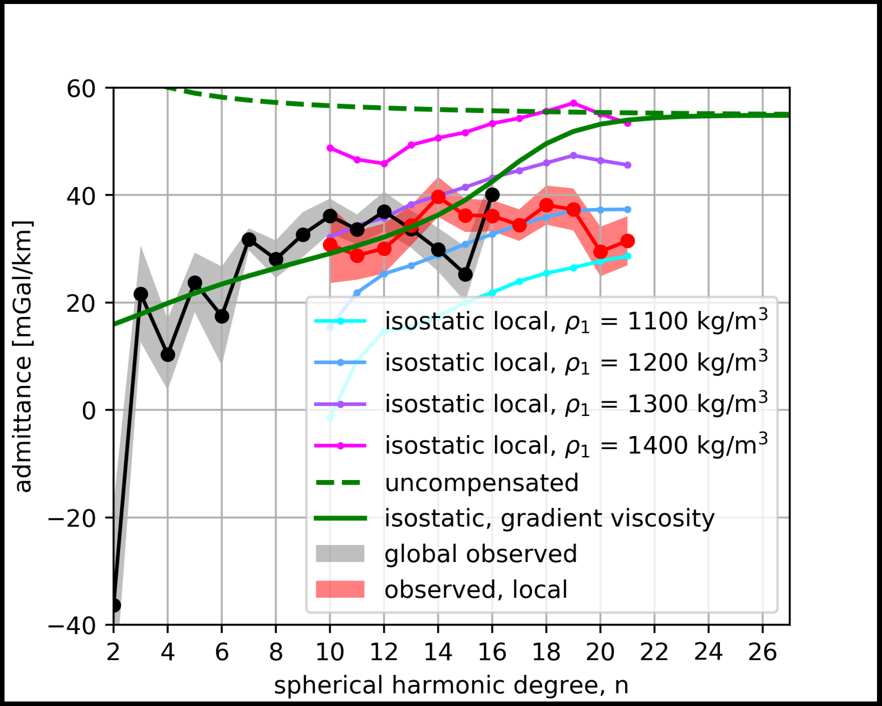




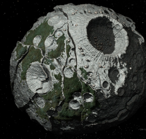
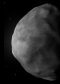
Admittance localized at high degree strength region

- **Localized admittance is consistent with isostatic compensation up to *n* = 21**

- **The mean crustal density is found to be 1233^{+45}_{-36} kg/m^3 from the Dawn XM2 data.**

- **Possible signature of vertical density gradient at high degrees**



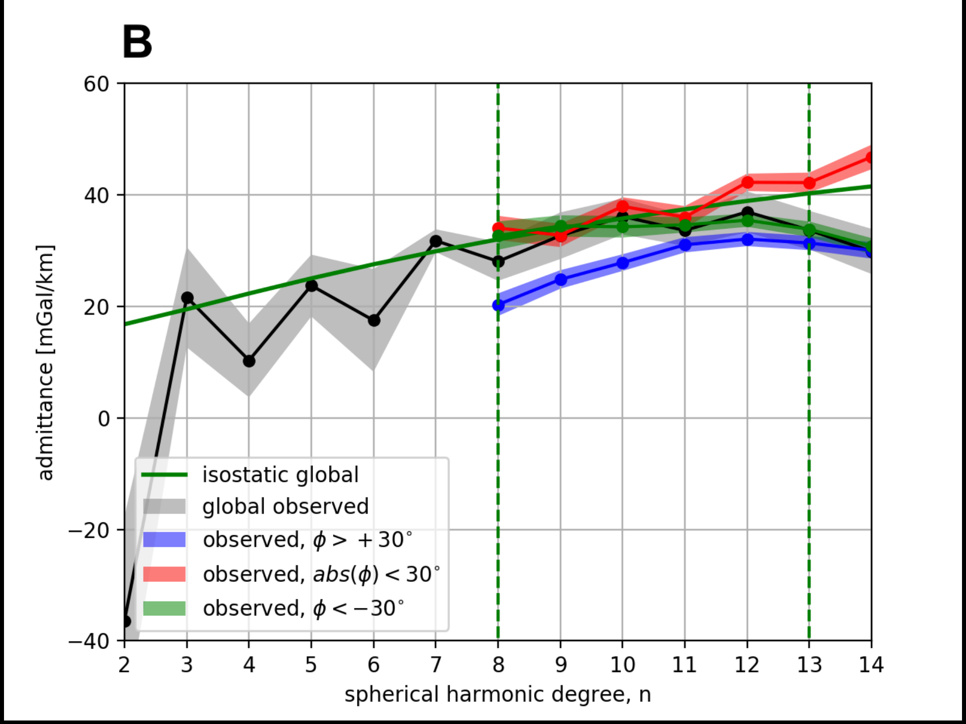
    

Summary

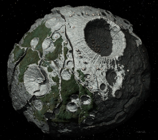
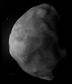
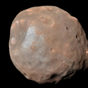
- Vesta uncompensated topography makes it possible to use effective density spectrum to constrain its density profile
  - Higher crustal density at the intersection of Rheasilvia and Veneneia basins -> the region of deepest impact excavation
  - Higher crustal density in the region of olivine detections
- Spectral-spatial localization of Ceres gravity and shape has not revealed the transition from compensated to uncompensated topography
  - This prevents using effective density spectrum for directly constraining crustal density of Ceres.
  - There remains a trade-off between compensation depth and crustal density.
  - Alternatively, a vertical density gradient in the crust of Ceres could be causing lower than uncompensated admittances

Global heterogeneities on Ceres revealed by admittance

- **Lower admittance near the poles, especially near the north pole**

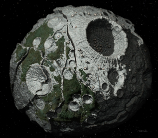
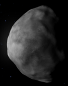
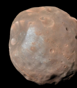




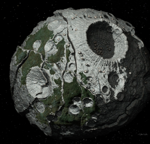
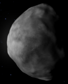
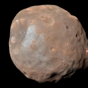
Eros



Future Psyche and Bennu



Effective density example

$$\boxed{\tilde{\rho}_n = \rho_{surface} + \frac{1}{R^{n+3}} \int_{R^-}^{0} r^{n+3} \frac{d\rho(r)}{dr} dr}$$

- **Assumptions:**
- **mean density**